An integrated media, integrated processes watershed model

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Abstract

Parametric-based, lumped watershed models have been widely employed for integrated surface and groundwater modelling to calculate surface runoff on various temporal and spatial scales of hydrologic regimes. Physics-based, process-level, distributed models that have the design capability to cover multi-media and multi-processes and are applicable to various scales have been practically nonexistent until late 1990s. It has long been recognized that only such models have the potential to further the understanding of the fundamental factors that take place in nature hydrologic regimes; to give mechanistic predictions; and most importantly to be able to couple and interact with weather/climate models. However, there are severe limitations with these models that inhibit their use. These are, among other things, the ad hoc approaches of coupling between various media, the simplification of modelling overland and/or river flow, and the excessive demand of computational time. This paper presents the development of an integrated media (river/stream networks, overland regime, and subsurface media), integrated processes (evaporation, evapotranspiration, infiltration, recharges, and flows) watershed model to address these issues. Rigorous coupling strategies are described for interactions among overland regime, rivers/streams/canals networks, and subsurface media. The necessities to include various options in modelling surface runoff and river hydraulics are emphasized. The options of selecting characteristic wave directions for two-dimensional problems are stated. The implementation of high performance computing to increase the computational speed is discussed. Four examples are used to demonstrate the flexibility and efficiency of the model as applied to a theoretical benchmark scale, a parallel computing, and two project-level large scale problems – one in Taiwan and the other in Florida.

1. Introduction

This paper presents the development of a numerical model simulating fluid flow in WAtersHed Systems of 1D Stream-River Networks, 2D Overland Regime, and 3D Subsurface Media (WASH123D). WASH123D is an integrated multimedia, multi-processes, physics-based computational model of various spatial-temporal scales [28,29]. The model was developed to have design capability to simulate flow in various component systems or combinations of component systems of a watershed. It can simulate problems of various spatial and temporal scales as long as the assumptions of continuum are valid. It has also been implemented with high performance computing in various versions to enable efficient applications to large problems [3–5]. Graphical pre- and post processors were developed in US Army Corps Groundwater Modelling System (GMS) to facilitate the application of the model to complex field problems.

1.1. Multimedia

WASH123D was developed to cover dendritic river/stream/canal networks and overland regime (land surface) (left plate of Fig. 1) and subsurface media including vadose and saturated (groundwater) zones (right plate of Fig. 1). It incorporates natural junctions and control structures such as weirs, gates, culverts, levees, and pumps in river/stream/canal networks. It also includes management structures such as storage ponds, pumping stations, culverts, and levees in the overland regime. In the subsurface media, management devices such as pumping/injecting wells, drainage pipes, and drainage channels are also included. Many management rules of these control structures and pumping operations have been implemented.

1.2. Multiprocesses

WASH123D is designed to deal with physics-based multi-processes occurring in watersheds. These include fluid flow over the entire hydrologic cycle (Fig. 2). The processes include (1) evaporation from surface waters (rivers, lakes, reservoirs, ponds, etc.) in
the terrestrial environment; (2) evapotranspiration from plants, grass, and forest from the land surface; (3) infiltration to vadose zone through land surface and recharges (percolations) to groundwater through water tables; (4) surface runoff (sheet flow) over the land surface; (5) hydraulics and hydrodynamics in dendritic rivers; and (6) subsurface flow in both vadose and saturated zones.

2. Theoretical bases of WASH123D

The theoretical bases of fluid flows built in WASH123D are based on the conservation laws of fluid and momentum with associated constitutive relationships between fluxes and state variables and appropriately formulated equations for source/sink terms. Various types of boundary conditions based on physics reasoning are essential to supplement the governing equations. Adequate initial conditions are either obtained from measurements or with simulations of steady-state versions of the governing equations.

The particular features in WASH123D are the inclusion of three approaches to model surface flow in a watershed system: the kinematic, diffusive, and dynamic wave models. The dynamic wave models completely describe water flow but are very difficult to solve under some conditions (e.g., when the slope of ground surface is steep), regardless of what numerical approach is employed. On the other hand, the diffusion and/or kinematic models can handle a wide range of flow problems but are inaccurate when the inertial terms play significant roles. Thus, three options are provided in WASH123D to accurately or efficiently compute water flow over a wide range of conditions. Another unique feature of WASH123D is to rigorously couple flow among river networks, overland regime, and subsurface media based physics. Both direct and indirect connections are included. Fully dynamic wave equations governing surface flows are cast in the characteristic form and accurate Lagrangian particle tracking algorithms were used to solve these equations.

2.1. Water flow in one-dimensional river/stream/canal network

The governing equations of water flow in one-dimensional river/stream/canal can be derived based on the conservation laws of water mass and linear momentum [19]. The law of mass conservation results in the Continuity Equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial s} = S_5 + S_R - S_E + S_I + S_1 + S_2$$

where $t$ is time [T]; $s$ is the axis along the curvilinear river/stream/canal direction [L]; $A$ is cross-sectional area of the river/stream [L$^2$]; $Q$ is flow rate of the river/stream/canal [L$^3$/T]; $S_5$ is the man-induced source [L$^3$/T/L]; $S_R$ is the source due to rainfall [L$^3$/T/L]; $S_E$ is the sink due to evapotranspiration [L$^3$/T/L]; $S_I$ is the source due to exfiltration from the subsurface media [L$^3$/T/L]; $S_1$ and $S_2$ are the source terms contributed from overland flow [L$^3$/T/L].

The law of conservation of linear momentum results in the Momentum Equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial VQ}{\partial s} = -gA \frac{\partial (Z_o + h)}{\partial x} + (M_s + M_R - M_E + M_I + M_1 + M_2) + \frac{Bz^* - Prz^*}{\rho}$$

where $h$ is water depth [L]; $V$ is river/stream/canal velocity [L/T]; $g$ is gravity [L/T$^2$]; $Z_o$ is bottom elevation [L]; $M_s$ is the external momentum–impulse from artificial sources/sinks [L$^3$/T$^2$]; $M_R$ is...
the momentum–impulse gained from rainfall \([L^3/T^2]\); \(M_e\) is the momentum–impulse lost to evapotranspiration \([L^3/T^2]\); \(M_i\) is the momentum–impulse gained from the subsurface due to exfiltration \([L^3/T^2]\); \(M_2\) and \(M_3\) are the momentum–impulse gained from the overland flow \([L^3/T^2]\); \(p\) is the water density \([M/L^3]\); \(B\) is the top width of the cross-section \([L]\); \(t^*\) is the surface shear stress \([M/T^2/L]\), which can be assumed proportional to the flow rate as \(t^* / \rho = k V^2\) where \(k = g r^2 R^{1/3}\) and \(R\) is the hydraulic radius \([L]\) and \(n\) is the Manning’s roughness. Depending on the simplification of the momentum equation, one can have three approaches: fully dynamic wave, diffusive wave, and kinematic wave.

### 2.1.1. Fully dynamic wave models

For the fully dynamic wave approach, all terms in Eq. (2) are retained. Under such circumstances, the governing equations can be expressed in the conservative form, the advection form, or the characteristic form. In this paper the characteristic form of the fully dynamic approach will be used as the main option because it is the most natural way and amenable to the advective numerical methods, e.g., the Lagrangian–Eulerian method. Written in the characteristic form, Eqs. (1) and (2) are cast as [28]:

\[
\frac{DV_c}{Dt} (V + c) \frac{\partial (V + c)}{\partial t} + (V + c) \frac{\partial (V + c)}{\partial x} = \frac{g}{C_0} (R_1 + R_2) \tag{3}
\]

\[
\frac{D(V - c)}{Dt} (V - c) \frac{\partial (V - c)}{\partial t} + (V - c) \frac{\partial (V - c)}{\partial x} = - \frac{g}{C_0} (R_1 + R_2) \tag{4}
\]

in which

\[
R_1 = \frac{1}{B} (S_5 + S_k - S_2 + S_1 + S_2) - V \frac{\partial A}{\partial t} \tag{5}
\]

\[
R_2 = -g \frac{\partial Z_0}{\partial x} + \frac{1}{A} \left[ V(S_2 + S_k - S_2 + S_1 + S_1) + (M_5 + M_k - M_k) + M_1 + M_1 + M_2 + B \frac{\partial c^2}{\partial t} - P \frac{\partial c^2}{\partial \rho} \right] \tag{6}
\]

and

\[
c = \sqrt{\frac{gA}{B}} \quad \omega = \int_0^h \left( \frac{g}{C_0} \right)^{1/2} ds \quad A(s, t) = A^h (h(s, t), t) \tag{7}
\]

where \(c\) is the wave speed and \(\omega\) is the transformed wave speed. Eq. (3) simply states that the positive gravity wave \((V + c)\) is advected by the speed \((V + c)\) while Eq. (4) states that the negative gravity wave \((V - c)\) is advected by the speed \((V - c)\). To emphasize the nature of non-prism cross-sections, \(A^h (h(s, t), s)\) is used to denote \(A(s, t)\), i.e., \(A(s, t) = A^h (h(s, t), s)\) and \(B(s, t) = B^h (h(s, t), s) = \partial A^h / \partial h\).

For transient simulations, the water depth (or water stage) and the cross-sectionally averaged velocity must be given as the initial condition. In addition, appropriate boundary conditions need to be specified to match the corresponding physical system.

The system of Eqs. (3) and (4) are identical to the system of Eqs. (1) and (2) on the differential level. They offer advantages in their amenability to innovative advective numerical methods such as the semi-Lagrangian scheme. Furthermore, the specification of boundary conditions is very straightforward. Only when the wave is coming into the region of interest, the boundary condition is required. For the wave that is going out of the region of interest, there is no need to specify a boundary condition.

At an upstream node, either two boundary conditions or one boundary condition are needed depending on if the flow is supercritical or critical/subcritical. At a downstream node, no boundary condition is needed if the flow is supercritical or critical and one boundary condition is needed if the flow is subcritical. On internal boundaries such as natural junctions and control structures of weirs, gates, culverts, and levees, mass or energy balance is explicitly enforced by solving a set of flux continuity and state variable continuity equations. For the internal sources/sinks, pumping and operation rules are simulated to ensure conservation.

#### 2.1.2. Diffusive wave models

In a diffusive approach, the inertia terms in the momentum equation is assumed negligible when compared with the other terms. By further assuming \(M_e = M_k = M_i = M_3 = 0\), we substitute the simplified momentum equation into the continuity equation to yield [9,28]:

\[
\frac{B}{\alpha} + \frac{\partial H}{\partial t} - \frac{\partial}{\partial x} \left\{ K \frac{\partial H}{\partial x} - K \frac{B t^2}{R g \rho} \right\} = S_5 + S_k - S_2 + S_1 + S_1 + S_2 \tag{8}
\]

\[
K = \frac{a AR^{2/3}}{n} \left[ 1 + \left( \frac{\partial Z_0}{\partial x} \right)^2 \right]^{-2/3} \left( \left| \frac{\partial H}{\partial x} + \frac{B \sigma^2}{R g \rho} \right| \right)^{1/2} \tag{9}
\]

where \(n\) is Manning’s roughness \([T/L1^{1/3}]\), \(a\) is a unit-dependent factor \((a = 1\) for SI units and \(a = 1.49\) for US Customary units) to make the Manning’s roughness unit-independent, \(R\) is the hydraulic radius \([L]\), and \(H = h + Z_0\) is the water stage.

To achieve transient simulations, either water depth or stage must be given as the initial condition. In addition, appropriate boundary conditions need to be specified to match the corresponding physical system. Boundaries can be classified as global boundaries, internal junction boundaries, and internal structure boundaries. On a global open boundary node, one boundary condition is needed. Any of the three types of boundary conditions can be specified for an open boundary node: (1) Dirichlet condition in which the pressure head is prescribed, (2) Cauchy condition in which the flux is prescribed or (3) rating curve condition in which the flux is a known function of the water depth. On a global closed boundary node, the flux is zero. On a junction, one mass balance equation and a number of energy balance equations (equal to the number of river reaches connecting to the junction) are employed to solve the stage at the junction and fluxes from reaches to the junctions. At a control structure such as weirs, gates, culverts, etc., the continuity of fluxes is imposed and the discharges through various types of structures have been formulated and reported elsewhere [16].

#### 2.1.3. Kinematic wave models

In a kinematic wave approach, all the assumptions for the diffusive wave approach are held. However, the velocity is given by

\[
V = \frac{a A}{n} \left[ 1 + \left( \frac{\partial Z_0}{\partial x} \right)^2 \right]^{1/2} \left( \left| \frac{\partial Z_0}{\partial x} + \frac{B \sigma^2}{R g \rho} \right| \right)^{1/2} \tag{10}
\]

Substituting Eq. (10) into Eq. (1) and using the definition \(Q = VA\), we obtain

\[
\frac{\partial A}{\partial t} + \frac{\partial VA}{\partial x} = S_5 + S_k - S_2 + S_1 + S_1 + S_2 \tag{11}
\]

It is noted that Eq. (11) represents the advective transport of the cross-sectional area, \(A\). It is an ideal equation amenable for numerically innovative advective transport algorithm.

To achieve transient simulations, either water depth or stage must be given as the initial condition. In addition, appropriate boundary conditions need to be specified to match the corresponding physical configuration. In a kinematic wave approach, boundary conditions are required only at upstream boundaries. An upstream boundary can be open or closed. On an open upstream boundary,
either the cross-sectional area (equivalent to water depth or water stage) or the flow rate is prescribed. The flow rate through a closed upstream boundary point is by default equal to zero.

2.2. Water flow in two-dimensional overland flow

The governing equations for two-dimensional overland flow can be derived based on the conservation law of water mass and linear momentum [23]. The law of conservation of fluid mass results in the Continuity Equation as

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} = S + R - E + I$$  \(12\)

where \(h\) is the water depth \([L]\); \(u\) is the velocity component in the \(x\)-direction \([L/T]\); \(v\) is the velocity component in the \(y\)-velocity \([L/T]\); \(S\) is the man-induced source \([L^3/T/L^2]\); \(R\) is the source due to rainfall \([L^3/T/L^2]\); \(E\) is the sink due to evapotranspiration \([L^3/T/L^2]\); and \(I\) is the source from subsurface media due to exfiltration \([L^3/T/L^2]\). It should be noted that \(uh = q_u\) is the flux in the \(x\)-direction \([L^2/T]\) and \(vh = q_v\) is the flux in the \(y\)-direction \([L^2/T]\).

The conservation principle of linear momentum along the \(x\)-direction results in the \(x\)-Momentum Equation as

$$\frac{\partial (uh)}{\partial t} + \frac{\partial (uh^2)}{\partial x} + \frac{\partial (vh)}{\partial y} = -g\frac{\partial (Z_o + h)}{\partial x} + \left( M_f^x + M_r^x - M_s^x - M_b^x \right) + \frac{\tau_x^1 - \tau_y^2}{\rho}$$  \(13\)

where \(Z_o\) is the bottom elevation of overland \([L]\); \(M_f^x\) is the \(x\)-component of momentum–impulse from artificial sources/sinks \([L^2/T^2]\); \(M_r^x\) is the \(x\)-component of momentum–impulse gained from rainfall \([L^2/T^2]\); \(M_s^x\) is the \(x\)-component of momentum–impulse lost to evapotranspiration \([L^2/T^2]\); \(M_b^x\) is the \(x\)-component of momentum–impulse gained from the subsurface media due to exfiltration \([L^2/T^2]\); \(\tau_x^1\) is the component of surface shear stress along the \(x\)-direction over unit horizontal overland area \([M/L/T]\); and \(\tau_y^2\) is the component of bottom shear stress along the \(x\)-direction over unit horizontal overland area \([M/L/T]\), which can be assumed proportional to the \(x\)-component velocity, i.e., \(\tau_x^1/\rho = \kappa|V|u\), where \(\kappa = g|\nabla h|^{1/2}\). The law of conservation of linear momentum along the \(y\)-direction results in the \(y\)-Momentum Equation as

$$\frac{\partial (vh)}{\partial t} + \frac{\partial (uhv)}{\partial x} + \frac{\partial (vh^2)}{\partial y} = -g\frac{\partial (Z_o + h)}{\partial y} + \left( M_f^y + M_r^y - M_s^y - M_b^y \right) + \frac{\tau_y^2 - \tau_x^1}{\rho}$$  \(14\)

where \(M_f^y\) is the \(y\)-component of momentum–impulse from artificial sources/sinks \([L^2/T^2]\); \(M_r^y\) is the \(y\)-component of momentum–impulse gained from rainfall \([L^2/T^2]\); \(M_s^y\) is the \(y\)-component of momentum–impulse lost to evapotranspiration \([L^2/T^2]\); \(M_b^y\) is the \(y\)-component of momentum–impulse gained from the subsurface media due to exfiltration \([L^2/T^2]\); \(\tau_y^2\) is the component of surface shear stress along the \(y\)-direction over unit horizontal overland area \([M/L/T]\); \(\tau_x^1\) is the component of bottom shear stress along the \(y\)-direction over unit horizontal overland area \([M/L/T]\), which can be assumed proportional to the \(y\)-component velocity, i.e., \(\tau_y^2/\rho = \kappa|V|v\).

2.2.1. Fully dynamic wave models

Eqs. (12)–(14) written in the conservative form are the governing equations for two-dimensional overland flow. Depending on the simplification of the momentum equations, one can have three approaches: fully dynamic wave, diffusive wave, and kinematic wave. For the fully dynamic wave approach, all terms in Eqs. (13) and (14) are retained. Under such circumstances, the governing equations can be expressed in the conservative form, advection form, or characteristic form. In this paper the characteristic form of the fully dynamic approach will be used as the main option because it is the most natural way and amenable to innovative advective numerical methods, e.g., Lagrangian–Eulerian method. Written in the characteristic form, Eqs. (12)–(14) become [28]:

$$\frac{DxW_1}{Dt} = \frac{\partial W_1}{\partial t} + u\frac{\partial W_1}{\partial x} + v\frac{\partial W_1}{\partial y} + s_1 = A_1$$  \(15\)

$$\frac{Dx\cdot ck_1W_2}{Dt} = \frac{\partial W_2}{\partial t} + (u + ck_1^2)\frac{\partial W_2}{\partial x} + (v + ck_1^2)\frac{\partial W_2}{\partial y} + s_2 = A_2$$  \(16\)

$$\frac{Dx\cdot ck_2W_3}{Dt} = \frac{\partial W_3}{\partial t} + (u - ck_2^2)\frac{\partial W_3}{\partial x} + (v - ck_2^2)\frac{\partial W_3}{\partial y} + s_3 = A_3$$  \(17\)

in which

$$\begin{align*}
\{ \frac{\partial W_1}{\partial t} \} & = \left\{ \begin{array}{l} k_1^2\partial u - k_2^2\partial v \\
(2\alpha e + k_1^2\partial u + k_2^2\partial u)/g \\
(-2\alpha e + k_1^2\partial u + k_2^2\partial u)/g \\
R_1/c + k_2^2R_2/g + k_1^2R_1/g \\
-R_1/c + k_2^2R_2/g + k_1^2R_2/g \\
\end{array} \right\} A_1 \\
\{ \frac{\partial W_2}{\partial t} \} & = \left\{ \begin{array}{l} k_1^2R_2 - k_1^3R_3 \\
(2\alpha e + k_1^2\partial u + k_2^2\partial u)/g \\
(-2\alpha e + k_1^2\partial u + k_2^2\partial u)/g \\
\end{array} \right\} A_2 \\
\{ \frac{\partial W_3}{\partial t} \} & = \left\{ \begin{array}{l} k_2^2R_2 \\
-g\frac{\partial (k_1^2\partial u - k_1^3\partial v)}{\partial t} \\
g\frac{\partial (k_1^2\partial u - k_1^3\partial v)}{\partial t} \\
\end{array} \right\} A_3 \\
\end{align*}$$  \(18\)

where \(c = \sqrt{gh}\) is the wave speed; \(k_1^2\) is the unit vector along the direction of the vorticity wave; \(k_2^2\) is \((k_1^2, k_2^2)^T\) is the unit vector along the direction of the positive gravity wave direction; \(-k_2^2\) is \((-k_1^2, -k_2^2)^T\) is the unit vector along the direction of the negative gravity wave. Eq. (15) states that the vorticity wave \(W_1\) defined by \(\partial W_1 = k_1^2\partial u - k_1^3\partial v\) is advected by the velocity \(V\). Eq. (16) states that the positive gravity wave \(W_2\) defined by \(\partial W_2 = (2\alpha e + k_1^2\partial u + k_2^2\partial u)/g\) is advected by the velocity \((V + c k_1^2)\) while Eq. (17) states that the negative gravity wave \(W_3\) defined by \(\partial W_3 = (-2\alpha e + k_2^2\partial u + k_2^2\partial u)/g\) is advected by the velocity \((V - c k_2^2)\).

In solving Eqs. (12)–(14) or Eqs. (15)–(17), the water depth \(h\), and the velocity components, \(u\) and \(v\), must be given initially or they can be obtained by simulating the steady-state version of Eqs. (12)–(14). In addition, appropriate boundary conditions need to be specified to match the corresponding physical system. The characteristics form of the governing equation offers great advantages over the conservative form in adapting appropriate numerical algorithms and in defining boundary conditions. Innovative hyperbolic numerical algorithms can be employed to approximate the system because each of the three equations is a decoupled advective transport equation of a wave. The determination of the number of boundary conditions that are consistent with physics is not straightforward when the conservative form of equations is used. However, when the characteristic form of equations is
used, the number of boundary conditions that conform to the physics can easily be determined. We demonstrate how boundary conditions are specified in the following. An overland boundary segment can be either open or closed. In the former case, the boundary condition for any wave is needed only when it is transported into the region of interest. When a wave is transported out of the region, there is no need to specify the boundary condition because internal flow dynamics due to this wave affects the boundary values of $u$, $v$, and $h$. In other words, external world will not affect the wave that is transported out of the region.

At an open upstream boundary segment, $n \mathbf{V} < 0$; thus the vorticity is always transported into the region from upstream. If $n c(k^2) < 0$, then $-n c(k^2) > 0$. Under such circumstance, $n (V + c(k^2)) < 0$ and $n (V - c(k^2))$ may be less than or greater than 0; thus the positive gravity wave is transported into the region and the negative gravity wave may be transported into the region or out of the region. Similarly, if $n c(k^2) > 0$, then $-n c(k^2) < 0$. Under such circumstance, $n (V - c(k^2)) < 0$ and $n (V + c(k^2))$ may be less than 0 or greater than 0; thus the negative gravity wave is transported into the region and the positive gravity wave may be transported into the region or out of the region. If $n c(k^2) = 0$, then all three waves are transported into the region. Based on the above discussions, it is seen that at an upstream open boundary, either all three waves are transported into the region or two waves are transported into the region. Thus, either three boundary conditions or two boundary conditions are needed. When three boundary conditions are needed, the water depth and two velocity components are prescribed. When two boundary conditions are needed, one of the boundary conditions is user’s specified water depth, normal flux, or rating curve flux and the other would be obtained by assuming the tangential flux is zero.

At an open downstream boundary segment, $n \mathbf{V} > 0$; thus the vorticity is always transported out of the region. If $n c(k^2) < 0$, then $-n c(k^2) > 0$. Under such circumstance, $n (V + c(k^2)) > 0$ and $n (V - c(k^2))$ may be greater than 0 or less than 0; thus the positive gravity wave is transported out of the region or into the region. Similarly, if $n c(k^2) > 0$, then $-n c(k^2) > 0$. Under such circumstance, $n (V - c(k^2)) > 0$ and $n (V + c(k^2))$ may be greater than 0 or less than 0; thus the negative gravity wave is transported out of the region and the positive gravity wave may be transported out of the region or into the region. If $n c(k^2) = 0$, then all three waves are transported out of the region. Based on the above discussions, it is seen that at an open downstream boundary, either all three waves are transported out of the region or only one wave is transported into the region. Thus, either no boundary condition or only one boundary condition is needed. When only one boundary condition is needed, its equation is the user’s specified water depth, normal flux, or rating curve flux.

At the closed upstream boundary, the normal flux must be zero, i.e., $[n \mathbf{V}] h = 0$. To satisfy this condition, three possibilities can occur: (1) both water depth and the normal component of the velocity are zero, (2) normal component of the velocity is zero and water depth is not zero, (3) water depth is zero and the normal component of the velocity is not zero. For Possibility (1), all three waves are standing and no boundary condition is needed. For Possibility (2), the vorticity wave is standing and one of the two gravity waves is transported out of the region while the other is transported into the region. Under such circumstance, only one boundary condition is needed, which is the normal component of the velocity equal to zero itself. For Possibility (3), all three waves are transported out of the region, no boundary condition is needed.

2.2.2. Diffusive wave models

In a diffusive approach, the inertia terms in the momentum equations is assumed negligible when compared with the other terms. By further assuming $M_1 = M_2 = M_3 = M_4 = M_5 = M_6 = 0$, we substitute the simplified momentum equation into the continuity equation to yield [28].

$$\frac{\partial h}{\partial t} + \nabla \cdot (h \mathbf{V}) = S + R - E + I$$

In which $H = h + Z_0$ and $K = \frac{ah^{5/3}}{n} \left[1 + (\nabla Z_0)^2\right]^{-2/3} \left(-\frac{\nabla H + \mathbf{t}}{\rho g h}\right)^{-1/2}$

where $n$ is Manning’s roughness [$L^{1/3}$], $a$ is a unit-dependent factor ($a = 1$ for SI units and $a = 1.49$ for US Customary units) to make the Manning’s roughness unit-independent, $R$ is the hydraulic radius [$L$], and $H$ is the water stage.

To achieve transient simulations, either water depth or stage must be given as the initial condition. In addition, appropriate boundary conditions need to be specified to match the corresponding physical system. Boundaries can be classified as open and closed boundaries. On an open boundary node, one boundary condition is needed. Three types of boundary conditions can be specified for any open boundary: (1) Dirichlet condition in which the water depth or stage is prescribed, (2) Cauchy condition in which the flux is prescribed, or (3) rating curve condition in which the flux is a known function of the water depth. On a closed boundary node, physics dictates that the normal flux is zero.

2.2.3. Kinematic wave models

In a kinematic approach, all the assumptions for the diffusive approach are held. However, the velocity is given by

$$V = -\frac{a}{n} \left[1 + (\nabla Z_0)^2\right]^{-2/3} \left[\nabla h - \frac{Bt^2}{\rho g h}\right]$$

Substituting Eq. (23) into Eq. (12), we obtain

$$\frac{\partial h}{\partial t} + \nabla \cdot (h \mathbf{V}) = S + R - E + I$$

It is noted that Eq. (24) represents the advective transport of the water depth, $h$. It is an ideal equation amena ble for numerically innovative advective transport algorithm. To achieve transient simulations, either water depth or stage must be given as the initial condition. In addition, appropriate boundary conditions need to be specified to match the corresponding physical configuration. In a kinematic wave approach, boundary conditions are required only at upstream boundaries. An upstream boundary can be an open or closed. On an open upstream boundary, either water depth or the flux can be specified. The flux through a closed upstream boundary point is by default equal to zero.
2.3. Water flow in three-dimensional subsurface media

The governing equation of subsurface flow through saturated–unsaturated porous media can be derived based on the conservation law of water mass [25]. It is written as follows:

$$F \frac{\partial h}{\partial t} = \nabla \cdot [K \cdot (\nabla h + \nabla z)] + \rho' q / \rho; F = a' \frac{\partial h}{\partial x} + \beta' \frac{\partial e}{\partial x} + n_e \frac{\partial S}{\partial h}$$  \hspace{1cm} (25)

where $\rho$ is the density of water; $h$ is the referenced pressure head [L]; $t$ is the time [T]; $K$ is the hydraulic conductivity tensor [L/T]; $z$ is the potential head [L]; $\rho'$ is the density of source water; $q$ is the source and/or sink [L$^3$/L$^2$/T]; $F$ is the water capacity [1/L]; $a'$ is the modified compressibility of the medium [1/L]; $\beta'$ is the compressibility of water [1/L]; and $S$ is the degree of saturation.

To achieve transient simulation, both initial and boundary conditions must be specified. The initial condition can be obtained either from field measurements or from simulation of the steady-state version of Eq. (25) with constant boundary conditions. Five types of boundary conditions can be prescribed for subsurface flows: (1) specified pressure head, (2) specified flux, (3) specified pressure gradient, (4) variable conditions in which the model will iteratively determine head or flux conditions (this type of boundary conditions is normally specified at the atmospheric boundary), and (5) radiation conditions where the flux is proportional to the difference in head between the media and surface waters such as rivers or lakes/reservoirs/ponds.

2.4. Coupling fluid flows among various media

One of the critical issues in a physics-based watershed model is its treatments of coupling among various media. There appear a number of watershed models that have dealt with each component medium on the bases of physics in the past decade (MIKE11-MIKE SHE [1,2], SHETRAN [7], MODFLOW-HMS [12], InHM [21], GISWA [24], SFRSM-HSE [17], COSFLOW [26], WASH123D Version 1.0 [27]). However, rigorous considerations on coupling among media seemed lacking. For example, an interface term is usually formulated between the river/stream/canal dynamics and subsurface flow (e.g., MODNET [22] or between overland and subsurface flows (e.g., MIKE11-MIKE SHE [26]). The linkage term usually introduces non-physical parameters. As a result, such watershed models have degraded even though each media-component module has taken a physics-based approach. A rigorous treatment of coupling media should be based the continuity of state variables and fluxes if the interface exhibits a direction connection. If the interface exhibits an indirect connection, then the continuity of fluxes should be imposed and a formulation of fluxes, such as a linkage term, must be proposed. Detailed mathematical descriptions of coupling between overland flow and river network flows, between river network and subsurface media, and between overland and subsurface flows can be found elsewhere [28]. The advantage and disadvantages of rigorous versus linkage coupling among media have been addressed [11].

3. Numerical methods

To provide robust and efficient numerical solutions of the governing equations, many options and strategies are provided in WASH123D so that a wide range of application-depending circumstances can be simulated. For surface flow problems, the semi-Lagrangian method (backward particle tracking) was used to solve kinematic wave equations. The diffusion wave models were numerically approximated with the Galerkin finite element method or the semi-Lagrangian method. The dynamic wave model in the characteristic wave form is numerically solved with the Lagrangian–Eulerian method. The governing equations of subsurface flow were discretized with the Galerkin finite element method. The dynamic wave model for surface water flows in conservative forms is discretized with the finite element method that satisfies the Ladhzenskaya–Bauska–Brezz (LBB) condition in the latest version of WASH123D.

When the fully dynamic wave option is numerically solved with the Lagrangian approach, one of the key issues is the selection of the directions of characteristic waves. For one-dimensional problems, the selection is straightforward since there is only one direction. For two-dimensional problems, there are infinite directions but only two independent directions are needed to solve well-posed problems. Thus, the key in the Lagrangian step is the selection of these two directions. Five options are provided in WASH123D: (1) both angles are specified by users, (2) first angle is along the gradient of the pressure and second angle is along the velocity, (3) first angle is along the gradient of the pressure and second angle is by diagonalization of the two gravity waves, (4) first angle is along the gradient of the pressure and second angle is other side of unite circle [8], and (5) both first and second angles are along the Froude line. When a convergent solution is achieved, all five options yield almost identical solutions. However, for some problems, some of these options have difficulties in achieving convergent solutions. These issues have been addressed elsewhere [10].

4. Example problems

The computer code, WASH123D, has been applied to many of the 68 projects in the world largest CERP (Comprehensive Everglade Restoration Project) program with cost of $8 billions. It has been chosen by the US Army Corps as the core computational code to model Lower East Coast (LEC) Wetland Watershed [3,4] including Biscayne Bay Coastal Wetland Watershed [13] and C111 Watershed in South Florida [14]. It has also been employed to construct a Regional Engineering Model for Ecosystem Restoration (REMER) that covers the eastern portion of Florida south of Lake Okeechobee [20]. A newly revamped WASH123D has been proposed to apply to many of river basins in Taiwan for flood and inundation forecast for the purposes of disaster reductions [18].

Four examples are presented in this paper. A benchmark problem [15] is given in Section 4.1 to illustrate the need of various options in modelling river flow. The high performance parallel version of WASH123D as applied to REMER is presented in Section 4.2. An example of applying the model to LanYang river basin is given in Section 4.3. An example to show the differences in coupling system components is given in Section 4.4.

4.1. Verification problems

Four benchmark problems were presented in MacDonald et al. [15]. Analytical solutions for the steady state flow with constant boundary and source conditions are available. Case 1 is a subcritical flow. Cases 2 and 3 are mixed subcritical and supercritical flow. Case 4 is a mixed subcritical and supercritical flow with a hydraulic jump. Detailed description of these benchmark case problems can be found in MacDonald et al. [15] and will not be repeated here. All cases were simulated with WASH123D using the fully dynamic wave model (DYW), diffusive wave model (DIW), and kinematic wave model (KIW). Only Cases 1 and 4 are presented here in Figs. 3 and 4, respectively.

It is seen that for the case of subcritical flow (Fig. 3) the DYW model yields very accurate results while DIW model has 4% error.
The KIW model renders unacceptable simulations. The DIW model generally demands much less computational time than the DYW model. Thus, it is preferable to use DIW model for real-world applications when the flow is subcritical. On the other hands, for research purposes, the accurate simulations are required; thus the DYW model should be used so the physics will not be distorted. When the flow is mixed subcritical and supercritical with hydraulic jumps, only the DYW model gives accurate results. Both DIW and KIW models generates unacceptable results (Fig. 4). Thus, for mixed subcritical and supercritical flows, the use of DYW models is mandatory even though its computational time may be excessive. The use of DIW or KIW models simply cannot capture the hydraulic jumps correctly. Therefore, it is important that a practical code should include all three model options for situation-dependent applications to a wide range of field problems.

4.2. High performance parallel computing examples

The example employed in this study is Sub-domain SD-5 of the modified REMER (Regional Engineering Model for Ecosystems Restoration) model (Fig. 5). SD-5 considered an area (~570 square miles) south of the Tamiami Trail in South Florida, which was bounded by Tamiami and C-4 canals in the north and coastal shores of the Gulf of Mexico, Florida Bay, and Biscayne Bay in the south. The model boundary was determined based on the available surface and groundwater gauge data, so that adequate boundary
conditions can be applied. Fig. 5 shows the topographical color contours in and around the computational domain adopted.

Three meshes of different resolutions were generated to test pWASH123D performance on the ERDC DSRC high performance computing (HPC) machine [6]. Table 1 lists the specification of the three meshes generated for this study (i.e., Coarse, Medium, and Fine). It is noted that the vertical spacing ranges from 2 ft for the top layers through several tens of feet for the middle and bottom layers depending on the thickness of the hydrogeologic units (e.g., aquifers, aquitards, and aquicludes) taken into account.

A total of 20 simulation runs were conducted on Sapphire, ERDC DSRC’s Cray XT3 (Table 2). Table 2 lists both the wall-clock time and PE (Parallel Efficiency) values for the Coarse, Medium, and Fine meshes. The PE for 2D computation has a value higher than 0.85 up

### Table 1
Specification of three meshes.

<table>
<thead>
<tr>
<th>Mesh ID</th>
<th>Submesh</th>
<th>No. of nodes</th>
<th>No. of elements</th>
<th>Average horizontal spacing (ft)</th>
<th>No. of 3D vertical elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>1D</td>
<td>214</td>
<td>186</td>
<td>3000</td>
<td>6</td>
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<tr>
<td></td>
<td>2D</td>
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<td>3D</td>
<td>59,409</td>
<td>99,498</td>
<td></td>
<td></td>
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<tr>
<td>Medium</td>
<td>1D</td>
<td>496</td>
<td>467</td>
<td>1250</td>
<td>12</td>
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<tr>
<td></td>
<td>2D</td>
<td>42,941</td>
<td>84,996</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3D</td>
<td>558,233</td>
<td>1,019,952</td>
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<td></td>
</tr>
<tr>
<td>Fine</td>
<td>1D</td>
<td>760</td>
<td>732</td>
<td>800</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2D</td>
<td>101,148</td>
<td>200,935</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>3D</td>
<td>2,124,108</td>
<td>4,018,700</td>
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</table>

### Table 2
Wall-clock time and parallel efficiency.

<table>
<thead>
<tr>
<th>Mesh ID</th>
<th>No. of processors</th>
<th>Over all</th>
<th>For 2D computations</th>
<th>For 3D computations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Wall-clock time (h)</td>
<td>PE</td>
<td>Wall-clock time (h)</td>
</tr>
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<td>0.9147</td>
<td>5.9611</td>
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<td></td>
<td>4</td>
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<td>0.3759</td>
<td>0.5270</td>
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<td>0.2218</td>
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<tr>
<td></td>
<td>64</td>
<td>3.0101</td>
<td>0.3556</td>
<td>0.2684</td>
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<tr>
<td>Middle</td>
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<td>76.1114</td>
<td>0.9334</td>
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<td>16</td>
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<td></td>
<td>64</td>
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<td>0.3005</td>
<td>1.3418</td>
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<td></td>
<td>128</td>
<td>15.9113</td>
<td>0.2959</td>
<td>5.2798</td>
</tr>
<tr>
<td>Fine</td>
<td>32</td>
<td>118.1105</td>
<td>0.9269</td>
<td>2.9920</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>63.7120</td>
<td>0.8860</td>
<td>2.4161</td>
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<td></td>
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<td>0.7986</td>
<td>2.2546</td>
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<td></td>
<td>96</td>
<td>49.2980</td>
<td>0.6335</td>
<td>2.1541</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>46.6115</td>
<td>0.4731</td>
<td>2.2691</td>
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<tr>
<td></td>
<td>192</td>
<td>41.6096</td>
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<td>2.4191</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>42.6953</td>
<td>0.1549</td>
<td>3.4590</td>
</tr>
</tbody>
</table>

Fig. 6. Topography and land use of Lanyang River Basin.
to 8, 32, and 80 processors for the Coarse, Medium, and Fine meshes, respectively (Table 2). The PE for 3D computation has a value higher than 0.85 up to 8, 64, and 96 processors for the Coarse, Medium, and Fine meshes, respectively (Table 2).

4.3. Modelling Lanyang river basin in Taiwan

The Langyang river basin is located in the northeastern Taiwan (left plate of Fig. 6). The area of this watershed is approximately 978 km². This area has a short hydrological response time due to its steep topography, varying from 3740 m in height on the hilltops to sea level at the entrance of the estuary [18]. Nine different kinds of surface usages are classified in the conceptual model of Lanyang river basin (right plate of Fig. 6). The surface elevation data for the study site are extracted from a digitized aerial photograph at 40-m intervals. For model calibrations and validations, the domain of interest is discretized with a triangular mesh of 6312 nodes and 12,209 elements for the 2D overland regime. For the 1D river network, it comprises of three reaches and one junction, which are discretized with 100 nodes and 97 elements. The model was successfully calibrated with two extreme typhoon events, Typhoon Area (August 23rd–26th, 2004) and Typhoon Nanmadol (December 3rd–4th, 2004). It was then validated with Typhoon Longwang (August 1st–2nd, 2005). The flash flood forecast results are shown in Fig. 7 [18]. For the mountain region, the diffusive wave option was used.

4.4. Dade county watershed modelling

This is a regional scale modelling effort for the South Florida wetlands. The Dade model domain extends from four miles west of the L-67 Extension dike to the western shore of Biscayne Bay.
and from one mile north of the Tamiami canal south to Florida bay. Vertically, it extends from the land surface to the bottom of the surficial aquifer. Some characteristics of this model are: (1) strong interaction of overland flow/groundwater flow and canal flow in south Florida; (2) complex hydraulic structure operations.

The 3D finite element mesh for subsurface media (Fig. 8) is complex: there are 37,760 global nodes, and 65,429 elements. There are seven layers in the vertical direction, and levees are incorporated as part of subsurface media. The boundary conditions for subsurface flow were determined from the SFWMM model output for the northern boundary, and from structure operation records for the other sides of boundaries. The 2D overland flow domain consists of 4720 nodes, and 9347 triangular elements. Levees are included in the computation domain (left plate of Fig. 9). Boundary conditions were determined from structure operation records along the boundary. The simplified canal network for this simulation includes: 560 Canal nodes, 506 Canal elements, 55 River reaches, 20 canal junctions, and 11 interior Gates (right plate of Fig. 9).

The combined processes of 1D canals, 2D overland flow, and 3D subsurface flow are complex and require a systematic modelling approach of adding computational complexity until all of the
important processes are simulated. This is primarily because the physical processes being simulated cannot all be turned on at the same time due to initial condition issues. It is important to start with consistent boundary and initial conditions and to run the model first in steady state 3D-only mode followed by 2D/3D overland and groundwater flow, followed by fully coupled 1D/2D/3D mode. In each step of the process, increasingly complicated but accurate initial conditions are provided to the next stage of the simulation until all physical processes are simulated.

Results from the coupled 2D overland flow and 3D subsurface flow simulations are voluminous due to the large mesh and the two-hourly time steps made over the one year time period. In this paper, only spatial distributions of model output at a discrete time are presented. Fig. 10 shows spatial distributions of simulation results for an early point in the yearlong simulation. The left plate shows coupled overland depths in the coupled 2D/3D simulation. The right plate shows coupled 2D/3D total head elevations of calibration.

After the model produced reasonable responses based on the coupled 2D overland and 3D subsurface flow simulation, initial conditions were saved and used as input to simulations of coupled 1D canal flow, 2D overland flow, and 3D subsurface flow. These simulations are the most rigorous of the coupled simulations. Fig. 11 shows spatial distributions of model output for an early
point in the yearlong simulation. The left plate shows overland flow depths in the coupled 1D/2D/3D simulation. The right plate shows coupled 1D/2D/3D total head elevations of calibration.

Comparing Figs. 10 and 11, we can see that there is a significant difference between 2D/3D and 1D/2D/3D simulations. It is thus important to consider interactions among all system components.

5. Conclusions

This paper presents the development of an integrated media, integrated processes watershed model. Theoretical base of the model are stated in terms of governing equations, boundary conditions and numerical approximations. Four examples are presented. The first one is used to verify the model and to illustrate the need of providing various wave options to deal with a wide range of problems either for accurate or efficient simulations. The second one evaluates the performance of parallel computing by applying the model to a large ecosystem restoration project. It shows that after a certain number of processors, the parallel efficiency falls below 0.5, which implies more processors than necessary do not increase the computational speed. The third one involves in the calibration of Lanyang river basin using the coupled 1D and 2D module of the model and demonstrates the applicability of the model in flood and inundation forecast in Taiwan. The fourth one applied to a large watershed in Florida. It demonstrated the need of a fully coupled model when interactions among river, overland, and subsurface flows are predominant.

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